CORRECTIONS TO "INERTIA GROUPS AND FIBERS"

BRIAN CONRAD

The regularity conclusion in Lemma 1.4 is incorrect (G. Pappas gave a counterexample). Regularity of X must be assumed as a hypothesis, and then the other conclusions follow (see below; also cf. [CEPT, Thm A1], which considers a related situation). As a consequence of this change in Lemma 1.4, normality hypotheses on schemes need to be replaced with regularity hypotheses throughout the paper (including the abstract). In particular, the concept of a *normal integral model* (U, f_U, i) as defined on the top of page 11 should be replaced everywhere by the concept of *regular integral model*, in which the source of f_U is also required to be regular. In the main case of interest, curves, this is essentially harmless. In the paragraph preceding Corollary 4.3, it must be assumed that X is *regular* (which is true away from finitely many closed points of S); this leads to some changes in Corollary 4.3, given below.

We now give the corrections to the proof of Lemma 1.4. Line 7, page 9 should read $\mathscr{B}_{X/Y} = \bigcup_i \mathscr{B}_{X_i/Y} = \bigcup_i D_i = D$, where the (reduced) support D_i of $\mathscr{B}_{X_i/Y}$ is a normal crossings divisor (by the normal crossings hypothesis on D and purity of the branch locus applied to $X_i \to Y$). The rest of the proof is OK, up to where we prove that $G \subseteq \prod \mu_{e_i}$ is trivial. By the *regularity* of Y' and $X = Y'/G$, the method of proof of [CEPT, Lemma A2] implies $G = \prod (G \cap \mu_{e_i})$. The original argument shows that the inertia subgroup $\mu_{e_i} \subseteq \mathrm{Aut}(Y'/Y)$ injects into the quotient $\mathrm{Aut}(X/Y)$, so $G \cap \mu_{e_i} = 1$ for all i and hence $G = 1$.

The original method of proof of Lemma 1.4, together with equation (1.1), does correctly prove a criterion for X to be regular if we only assume X is normal: for $x \in f^{-1}(y)$, G_x the Galois group of the generically Galois abelian covering $\mathrm{Spec}(\mathscr{O}^{\mathrm{sh}}_{X,x}) \to \mathrm{Spec}(\mathscr{O}^{\mathrm{sh}}_{Y,y})$, and $\{I_1, \dots, I_r\}$ the inertia subgroups of G_x at the generic points of the branch locus, X is regular at x if and only if $I_1 \times \cdots \times I_r \to G_x$ is injective. This is used in the special case $r = 2$ in the proof of the corrected form of Corollary 4.3 below.

We need to modify the statement of Corollary 4.3 to account for the possibilities that X might not be regular and that some $\overline{\{a_i\}}$ might not be geometrically unibranched at its closed points. The first two conditions in the definition of $\{s_1, \dots, s_n\}$ in Corollary 4.3 should be replaced with the single condition:

- $\bigcup \overline{\{a_i\}}$ is either not a normal crossings divisor at some y over s, or is a normal crossings divisor at some such y with base change to $\mathscr{O}^{\mathrm{sh}}_{Y,y}$ having two irreducible components with non-trivially intersecting inertia groups in some factor field of $K(X_K) \otimes_{K(Y_K)} \mathscr{O}^{\mathrm{sh}}_{Y,y}$.

The first condition in Corollary 4.3(2) should be replaced with

- for some $y \in Y$ over s, three of the $\overline{\{a_i\}}$'s meet at y, or two of these meet with non-reduced intersection at y, or two of these meet with reduced intersection at y and have non-trivially intersecting inertia groups in some factor field of $K(X_K) \otimes_{K(Y_K)} \mathscr{O}^{\mathrm{sh}}_{Y,y}$

(and delete the parenthetical remark following Corollary 4.3(2)). Following Corollary 4.3, we need to add:

- the set of $s \in S$ such that exactly two sections $a_i, a_j \in Y(S)$ meet at some y over s and the codimension 1 generic points of these sections have non-trivially intersecting inertia groups in some factor field of $K(X_K) \otimes_{K(Y_K)} \mathscr{O}^{\mathrm{sh}}_{Y,y}$.

By excellence arguments, $\mathscr{O}^{\mathrm{sh}}_{Y,y}$ above can be replaced with the fraction field of its completion.

References

[CEPT] T. Chiburg, B. Erez, G. Pappas, M.J. Taylor, *ε-constants and the Galois module structure of deRham cohomology*, Annals of Mathematics **146** (1997), pp. 411–473.

Department of Mathematics, Harvard University, Cambridge, MA 02138, USA
E-mail address: bconrad@math.harvard.edu